SOLAR PANELS AND OPTIMIZATION

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INTRODUCTION

Solar Power, Inc. will be exporting solar panels to the island of Suluclac, which lies in the Pacific Ocean near the equator. The orientation of these panels needs to be optimized for solar collection throughout the year. The optimizations apply to both a daily time scale and an annual time scale.

To analyze and solve this problem, Mathematica[1] was used to visualize the optimization plots, and to find certain critical points. The Mathematical Visualization Toolkit[2] numerical root finder was used to find other critical points, and for making contour plots. Standard graphing calculators were also used.

This report explains the various optimizations that were found. The weather (as far as cloudiness is concerned) on a daily and annual basis on Suluclac is explained. The best period of the year for taking down the panels for maintenance is explained. Instructions on when to adjust the panels throughout the year are given. All the reasons and justifications are given and developed through the report.

BACKGROUND

The solar radiation falling on a tilted solar panel depends on several factors, including (but not limited to) the orientation of the plane with respect to the tangent plane of the earth, the time of the day, the season, and the weather. In order to optimize the sunlight that reaches the panels, these things must be understood and taken into account.

Plane Orientation

A solar panel can be oriented in any direction. However, for practical purposes, it should only be orientated so that it faces upwards toward the sun (otherwise, why have it there?). Also, for purposes here, it will only be allowed to be inclined in a southward direction. This is convenient for simplicity's sake.

Time of Day

Since the panels are only allowed to tilt up in a southward direction, then they only can get direct sunlight around noontime. If they were allowed to rotate around the roll axis then they could be optimized for all of the daylight hours in a day.

Seasons

The earth's axis of rotation is tilted off the tangent of its orbital plane by as much as (approximately) 23 degrees both north and south throughout its orbit. This occurs because the axis of rotation continuously points at the same general spot in space. As a result, the quantity of solar radiation hitting the earth at a given point varies as a function of the earth's position in its orbit. This is what causes seasons. As a result, the season directly affects the amount of solar radiation that the panels receive. The tilt of the earth is related to the seasons in Table 1. These tilts are cyclic, of course, over the time frame of a year.

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[1] Wolfram Research, Inc., Mathematica Version 3.0, http://www.wolfram.com
[2] http://amath.colorado.edu/java/MVT/packages/MathTool.html

Table 1: Seasons and the Earth's respective tilt, s.

Date(s)	~Degrees of tilt, s	Radians of tilt, s
December 22	-23	-.4
December 23 - March 21	-23 → 0 (increasing with time in this period)	-.4 → 0
March 22 - June 21	0 → 23 (increasing with time in this period)	0 → .4
June 21	23	.4
June 21 - September 22	23 → 0 (decreasing with time in this period)	.4 → 0
September 23 - December 22	0 → -23 (decreasing with time in this period)	0 → -.4

Weather

The weather, or more specifically the "cloudiness", directly affects the amount of solar radiation that reaches the panels. As more clouds form in the sky, the sun is blocked and the panels see less radiation. Likewise, fewer clouds allow more radiation to reach the panels. From meteorologist data, the cloudiness on Suluclac can be described by the following equation:

$$C(s,t) = \frac{3-(1+s)\cos^2 t}{3} \qquad [1]$$

where: C is the cloudiness,
s is the Earth's tilt with respect to its orbital plane [radians],
t is an angle proportional to the time of day [radians].

Energy

The amount of energy collected by the solar panels each day has been described by a physicist as

$$W(s,u) = (1-0.36s-1.1s^2+0.36s^3+0.34s^4)\cdot\cos(u)+(-1.3s+0.38s^2+1.2s^3-0.26s^4)\cdot\sin(u) \qquad [2]$$

where: W is the energy absorbed [kW h/(m^2 day)]
u is the tilt of the panel as compared to the plane of the Earth [radians].

The energy equation given above, has already incorporated the effects due to absorption and cloudiness, and is complete by itself.

DISCUSSION

The following subsections discuss the methods used and the outcomes of those methods in determining the optimizations for the solar panels on the island. All calculations were done using radians, but some graph axes were plotted in both degrees and radians for the sake of clarity.

Weather

The weather on Suluclac is not constant. It is, however, predictable. Equation 1 allows for the determination of the weather in both a daily and a yearly manner. Figure 1 shows the cloudiness plot of Equation 1 in terms of s and t. As this represents "cloudiness", a lower value means less clouds and hence more sunshine radiating the solar panels. Here, the Earth's tilt, s, varies from 23 degrees to –23 degrees. This correlates (see Table 1) to a six month period. But, since the Earth's orbit is (nearly) circular there is a cyclic pattern. In order to describe a whole year, one must start at the 23-degree point (for example) and trace out a path to –23 degrees, returning to the 23-degree point[3]. The angle, t, of the sun in relation to the panel varies with the time from 0600 hours to 1800 hours. This is when there is daylight on the island.

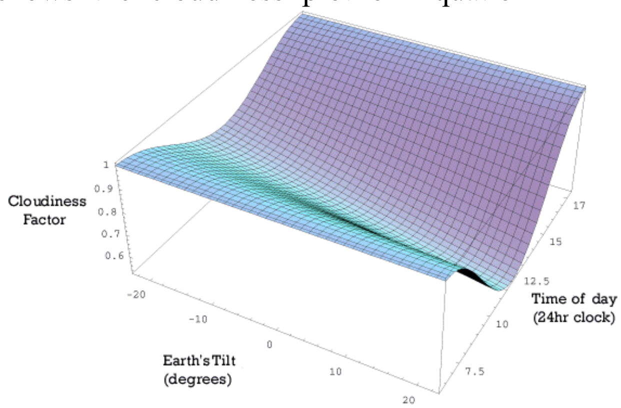

Figure 1: Cloudiness as a function of s and t.

Figure 1 shows that on any given day, the mornings and evenings are the cloudiest periods. In the middle of the day, there are very few clouds. Although the cloudiness in the mornings and evenings is the same throughout the year, the cloudiness throughout the rest of the day varies with the season. As Figure 1 shows, the days in the late spring and early summer are less cloudy than the days in the late fall and early winter.

Maxima and Minima

The result of determining optimal positioning of the solar panels is that the maximum energy can be obtained from them during any give period in the year. This means less solar panels are required, resulting in less overhead costs.

To find the maximum energy obtained the energy equation, Equation 2, was plotted (Figure 2). The plot height represents the amount of energy obtained from the solar panels with a variable panel tilt, u, variable throughout the Earth's range of tilt, s. The maximums and minimums either exist at *critical points* on the interior of the plot, or at maximum or minimum points on the plot's boundaries.

The partial derivatives of Equation 2 (Equations 3 and 4 below) were taken with respect to s and with respect to u.

[3] This correlates to a period starting on June 22, going to Dec 22, and ending on June 21.

$$\frac{\partial W}{\partial s} = (-.36 - 2.2s + 1.08s^2 + 1.36s^3)\cos u + (-1.3 + .76s + 3.6s^2 - 1.04s^3)\sin u = 0 \qquad [3]$$

$$\frac{\partial W}{\partial u} = -(1 - .36s - 1.1s^2 + .36s^3 + .34s^4)\sin u + (-1.3s + .38s^2 + 1.2s^3 - .26s^4)\cos u = 0 \qquad [4]$$

Using a contour plot (Figure 3) of the same region as in Figure 2, initial guesses were found and used in the numerical root finder to find the critical points on the interior of the plot. Only one exists at (s,u) = (-.353705, .430968). Using Equation 3, u was held constant at u = 0 and at u = 1 in order to find any critical points along those boundaries. One existed at u = 0 at s = -1.5423. None existed at u = 1. Using Equation 4, s was held constant at s = -0.4 and at s = 0.4. One critical point existed at (s,u) = (-.4, .48071). The boundary at u = 0.4 had no critical points. Finally, the corner points were checked. Only the values could be checked at these points, since none of them are critical points themselves. These 4 points, along with the other critical points found, are tabulated in Table 2 for easy digestion.

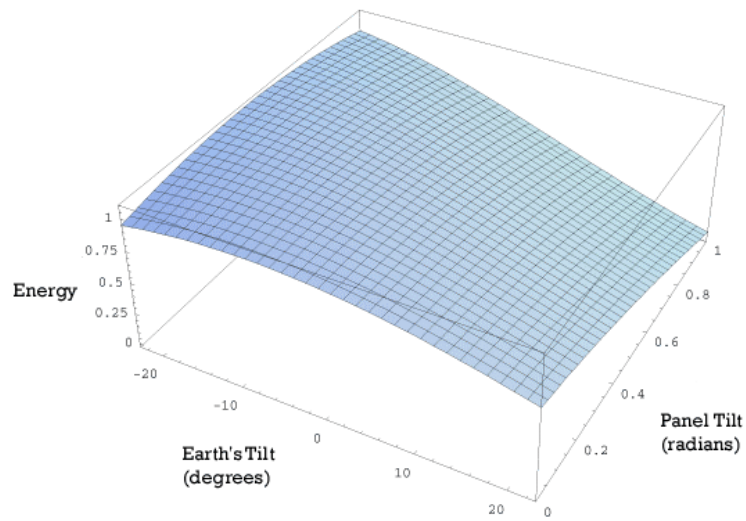

Figure 2: The energy surface plot.

Table 2: Critical points, and local maximums and minimums.

Location	Point (s, u)	Energy values	Local max or min
Surface	(-.353705, .430968)	1.07765	max
u = 0 boundary	(-.15423, 0)	1.02823	max
u = 1 boundary	None	n/a	n/a
s = 0.4 boundary	None	n/a	n/a
s = -0.4 boundary	(-0.4, 0.48071)	1.07556	max
Corner a	(-0.4, 0)	0.95366	neither
Corner b	(0.4, 0)	0.71174	neither
Corner c	(0.4, 1)	0.05718	min
Corner d	(-0.4, 1)	0.93377	neither

Since Table 2 contains all possible real values for maximum and minimum points, it is clear that the point (s_{max}, u_{max}) = (-.353705, .430968) on the surface of the plot is where the absolute maximum energy value of 1.07765 [kW hr/(m^2day)] can be obtained. In other words, the maximum amount of energy that can be collected by the solar panels happens roughly around late November or Early December, and also again in late January or early February when the solar panels are inclined at an angle of u_{max} = .430968 radians, or ~24.7degrees. Similarly, corner c is where the absolute minimum of .05718 [kW hr/(m^2day)] occurs. This minimum collection of energy will occur on or around June 22 every year with a panel inclination of u = 1 radian, or ~57.3 degrees.

Panel Service

By looking at Figure 2 and Figure 3, one can see that the time period with the lowest energy absorption (given a constant panel angle u) is the period around June 22, or 0.4 radians of Earth tilt. This is because June 22 is the point farthest from the date corresponding to the maximum point on the contour plot, and because the plot only decreases from that maximum point.

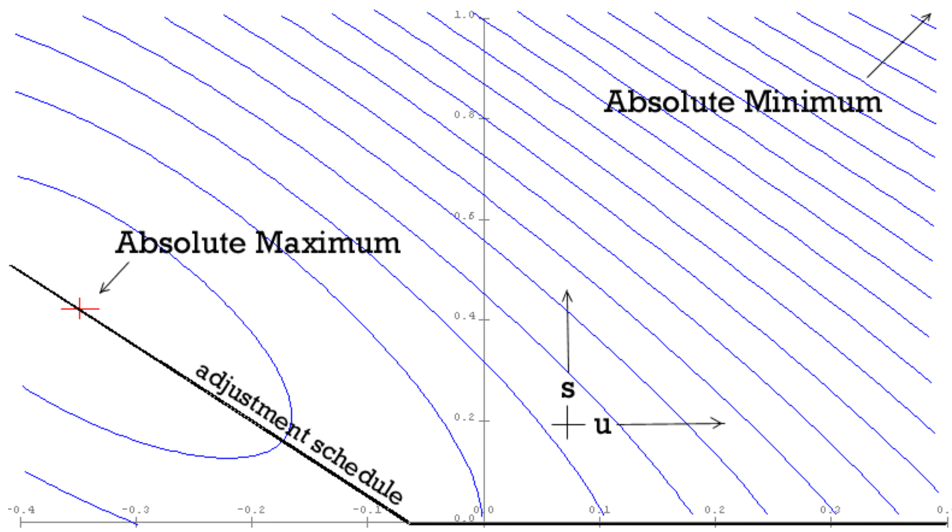

Figure 3: Contour plot of energy

Because annual maintenance requires an entire month to accomplish, it should be performed starting June 7, and finished July 7. This will keep the panels down during the time when they will lose the least amount of energy.

Weekly Panel Alignment

There will be a person employed to look after the solar plant on Suluclac on a daily basis. He or she will need to adjust the solar panel approximately once per week. The adjustments must be made in order that the maximum energy is absorbed by the solar panels. The *adjustment schedule* shown in Figure 3 can be followed to this end. For most of the year, the panels can be left in their zero degree inclination (u = 0) position. Starting in the fall and ending in the spring, however, the panels must be adjusted per the schedule.

Model Improvement

This model could be improved in several different ways. First, exact numbers instead of estimates could be used in all of the calculations. An example is using 23.5 degrees of tilt for the earth, instead of the 23-degree estimate. Second, exact dates could be calculated, instead of giving times in terms of general seasons. This would be much more useful to personnel working on the island. The Taylor Series polynomial coefficients to the sine and cosine functions in the energy equation could also be carried out farther to obtain more accuracy in the equation.

CONCLUSION

The problem of optimizing solar radiation collection by solar panels was studied. The use of calculus allowed the solution of this problem to be found. The reader should take from this report a clear idea of how to optimize solutions for real-world problems.

Final results were obtained for this problem. The absolute maximum and absolute minimum for the energy equation were found to be $(s_{max}, u_{max}) = (-.353705, .430968)$ and $(s_{min}, u_{min}) = (-0.4, 1)$, respectively. The best time for maintenance was also determined. This best time is at the turn of spring to summer. Also, by following the maintenance schedule on Figure 3, the solar cells can be adjusted to continuously absorb the maximum amount of energy available.